DOCUMENT RESUME

ED 184 880 SE 030 552

AUTHOR	Moser, James M.
TITLE	A Longitudinal Study of the Effect of Number Size and Presence of Manipulative Materials on Children's Processes in Solving Addition and Subtraction Verbal Problems.
SPONS AGENCY	National Inst. of Education (DHEW), Washington, D.C.
PUB DATE	Apr 80
GRANT	NIE-3-80-0117
NOTE	41p.; Paper presented at the Annual Meeting of the American Educational Research Association (Boston, MA, April 7-11, 1980). Contains light and broken type.
EDRS PRICE	MF01/PC02 Plus Postage.
DESCRIPTORS	Addition; Cognitive Development; *Cognitive Processes; Elementary Education; *Elementary School Mathematics; Interviews; *Longitudinal Studies; *Manipulative Materials; Mathematics Instruction; Problem Solving; Subtraction
IDENTIFIERS	*Mathematics Education Research

ABSTRACT

Strategies used by 150 children as they solved addition and subtraction verbal problems were studied over a period of one-and-a-half years. Number size and presence of manipulative materials were variables manipulated in individual interviews given at three separate intervals. Two addition and four subtraction tasks were used. Processes changed from primitive counting and modeling behaviors to more sophisticated strategies and use of number facts and heuristics. Presence of manipulatives tended to induce children to use simpler and less efficient processes. When confronted with larger numbers and lack of manipulatives, children used or invented more sophisticated strategies. (Author/MK)

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A Longitudinal Study of the Effect of Number Size and Presence of Manipulative Materials on Children's Processes in Solving Addition and Subtraction Verbal Problems

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Paper presented at the annual meeting of the

American Educational Research Association

Boston, Massachusetts

April 1980

The project presented or reported herein was performed pursuant to a grant from the National Institute of Education, Department of Health, Education, and Welfare. However, the opinions expressed herein do not necessarily reflect the position or policy of the National Institute of Education, and no official endorsement by the National Institute of Education should be inferred.

Center Contract No. NIE-G-80-0117

SE 030 552

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I. Introduction

No argument need be made about the importance of learning addition and subtraction skills. The "back to basics" movement is an indication of the general public's concern for computational expertise, of which addition and subtraction are an integral part. A cursory inspection of primary grade mathematics instructional materials is sufficient to confirm the emphasis placed upon these two basic operations. In recent years psychologists and mathematics educators have mirrored this interest by their empirical and theoretical investigations into the early learning of addition and subtraction. An important manifestation of this renewed research interest in addition and subtraction was the assembly of a group of international scholars at the Wingspread Conference, sponsored in November 1979 by the Wisconsin Research and Development Center for Individualized Schooling. It is the intent of this paper to report on a portion of the research work on addition and subtraction being carried out by the Mathematics Work Group of the Wisconsin Center.

Addition and subtraction problems presented to young children can be grouped into two categories. One category is the purely mathematical presentation of symbolic sentences in either horizontal or vertical form. The other category is nonsymbolic problems in which the numbers are measures of entities described in the problem situation. Relational and operational symbols can be used to represent the actions or relationships bearing upon those entities. Included in this second category are the so-called "story problems" encountered by youngsters in school. This report will deal only with problems in the second category.

Both types of problems have received attention from researchers. Weaver (1971) studied children's ability to solve a variety of horizontal open sentences where the "box" representing the unknown appeared in either the a, b, or c positions in the sentences $a \pm b = c$ and $c = a \pm b$. Numbers were

arranged so that some sentences had no whole number solution. Ability to correctly solve the open sentences varied from sentence type to type and among the grade levels tested. A similar study by Grouws (1972) was carried out. He also examined methods used by children to solve open sentences with slightly larger numbers than those used by Weaver. Other attempts at identifying methods used by subjects to solve open sentences were made in several response latency studies in which researchers matched response times to linear models in order to hypothesize use of certain counting models. (Groen & Parkman, 1972; Groen & Poll, 1973; Woods, Resnick & Groen, 1975). Recently, Lindvall & Ibarra (1980) have examined errors exhibited by first and second graders while solving certain horizontal open sentences.

Much of the work with nonsymbolic or verbal problems can be characterized by attempts to analyze on an apriori basis various components of verbal problems. The logical component was examined by Jerman & Rees (1972), Resnick, Wang & Kaplan (1973) and Nesher (1976). Syntax variables were considered by Jerman (1973), Jerman and Sanford (1974), Linville (1976), Searle, Lorton & Suppes (1974) and Nesher & Teubal (1974). Most relevant to the research to be reported in this paper are the efforts at semantic analysis of verbal problems. Perhaps the most thorough analysis is the work done by the French psychologist Vergnaud. He has looked at the complete range of problems having an additive structure including composition of additive transformations and problems involving negative numbers. (Vergnaud & Durand, 1976; Vergnaud, Note 1). Other semantic analyses have been carried out by Greeno and his associates at the University of Pittsburgh (Greeno, 1979; Heller & Greeno, Note 2; Riley & Greeno, Note 3), by Nesher & Katriel (Note 4) and by members of the Mathematics Work Group at the Wisconsin R & D Center (Moser, Note 5; Carpenter & Moser, Note 6).

A number of studies have examined children's performance on nonsymbolic addition and subtraction problems. Kindergarten children were studied by Grunau (1978), Ibarra and Lindvall (Note 7), Schwartz (1969) and Williams (1965). All of these considered only difficulty indices and did not examine strategies used by the respondents. Among other kindergarten studies that did investigate strategies used were those by Ginsburg and Russell (Note 8), Hebbeler (1977) and Hendrickson (1979). Primary grade children's behavior when presented with various types of verbal problems has been studied by Hirstein (1979), Steffe, Spikes & Hirstein (Note 9) and Riley (Note 10) where again the major point of interest was correctness rather than a direct observation of strategies used by the children.

The research to be reported in this paper has as its major focus the strategies used by children of primary school age. As such it differs from many of the work cited above. The research also has the characteristic of being longitudinal, having begun in Fall 1978 and still being carried cut as of the present time. The longitudinal study builds upon earlier pilot studies (Carpenter, Hiebert & Moser, Note 11; Carpenter, Note 12) and the semantic analyses described above (Moser, Note 5). Related data from the longitudinal study have been presented elsewhere (Carpenter & Moser, Note 6; Carpenter & Moser, Note 13).

II. Method

Subjects. In Fall 1978 all first-grade children from three schools in the Madison, Wisconsin area were candidates for inclusion in the study. All three schools serve a predominantly white, middle- to upper middle-class neighborhood. School 1 is a public primary school (Grades K, 1, and 2) with three first-grade teachers. School 2 is a large public school serving grades K through 8 with three first-grade teachers. School 3 is a smaller parochial school serving grades K through 8 with two first grade teachers. Inclusion in the study was finally determined on the basis of using as subjects only those children who returned parental permission slips allowing participation. Table 1 presents information about subjects.

Table 1

Subjects in longitudinal study

	School 1	School 2	School 3	Total
Number of children	59	50	35	144
Mean age	6:6	6:6	6:8	6:6
Maximum age	7:3	7:2	7:3	7:3
Minimum age	5:10	5:11	6:0	5:10
Males	32	26	24	82
Females	27	24	11	62

Results to be presented later were based upon interviews given at regular time intervals. The data in Table 1 describe the population interviewed at Interview 1. At Interviews 2 and 3, six subjects were added for a total of 150 children. Interview 3 occurred at the end of the first school year in May 1979. At that time, School 2 elected to participate no further in the study. That decision together with some natural attrition and addition caused by relocations of families resulted in 99 children participating in Interview

4 and 96 children in Interview 5. During the second year, the children from School 1 were assigned to classrooms served by four different teachers rather than the three in the first year.

Instruction during the course of the study. All schools use the Developing Mathematical Processes (DMP) program developed by the Wisconsin Research and Development Center for Cognitive Learning (Romberg, Harvey, Moser & Montgomery, 1974). Instructional units (or Topics as they are labeled in DMP) dealing explicitly with symbolization of addition and subtraction, with representation of addition and subtraction verbal problems by horizontal and vertical sentences, and with practice of "basic fact" combinations were not those appearing in the commercial version of DMP. Earlier evaluative data on DMP had indicated that some revision was in order. Prior to the commencement of the longitudinal study, a decision was made to produce six new topics dealing with the symbolic aspects of addition and subtraction. These six "S" topics were used by all classes, with Topics S1, S2, and S3 occurring in the first year of the study with first-grade children and Topics S4, S5, and S6 in the second year with second graders. The content of these six topics is described in detail in Kouba & Moser (Note 14).

These six topics along with the other regular DMP topics that provided the instructional experiences of the subjects of the study all have a very strong problem solving emphasis. Mathematics is approached through measurement with the notion that the measure of a set, object, or event is the way of quantitatively representing those entities. Relationships, and actions on and between these entities are then represented by a mathematical sentence. All of the problem situations presented to the subjects in the interviews are also covered at one time or another during instruction.

Interview tasks and procedures. Earlier semantic analyses indicated that a major dimension upon which verbal problems can be classified is whether or not the structure of the problem involves action or no action. In an action situation there is a change-causing action over a period of time that results in an increase or decrease in the size of some entity. In contrast there can be a static relationship existing at some fixed point in time among some entities. The action vs. static dimension was a major factor in the six verbal problems used in the study. Table 2 lists the six problems used in the order in which they were read to the subjects during the individual interviews.

Table 2

Verbal problems (tasks) used in longitudinal study

Task #	Problem label	Problem wording
1	Joining [addition]	Wally had 3 pennies. His father gave him 6 more pennies. How many pennies did he have altogether? (This is an action situation.)
2	Separating [subtraction]	Marie had 7 candies. She gave 5 candies to Fran. How many candies did Marie have left? (This is an action situation.)
3	Part-Part-Whole, missing addend [subtraction]	There are 5 jars of paint. Three jars are red and the rest are blue. How many jars of blue paint are there? (This is a static situation.)
4	Part-Part-Whole [addition]	Carol has 4 old shirts. She also has 7 new shirts. How many shirts does Carol have altogether? (This is a static situation.)
5	Comparison [subtraction]	Patrick has 9 fish. His sister Jill has 14 fish. How many more fish does Jill have than Patrick? (This is a static situation.)
6	Joining, missing addend [subtraction]	Fred has 8 M&M's. How many more M&M's does he have to put with them so he has 14 M&M's altogether? (This is an action situation.)

Problem tasks are characterized as addition or subtraction, not according to their semantic structure, but rather according to whether the operation of addition or subtraction upon the two given numbers in the problem yields the correct solution. At each of the different interviews and at each of the four conditions for an interview (to be described later) nouns and verbs within the problem wording were systematically changed so that children would not recognize the problem situations. Nouns and verbs were chosen, however, to preserve the semantic structure.

Within the problems, two of the three numbers from the number triple (x, y, z) defined by $x + y = z$, $x < y < z$ are given. In smaller number problems, $5 \leq z \leq 9$. In larger number problems, $11 \leq z \leq 15$. In the two addition tasks (Tasks # 1 and 4) the smaller addend x is given before the larger addend y. In the four subtraction problems, the larger addend y is given along with the sum z. In subtraction tasks # 2 and 3, the order in which these two numbers appear in the problem is (z, y) while in the other two subtraction tasks # 5 and 6 the order of presentation is (y, z). From the possible number triples for the smaller number domain and the larger number domain, six triples in each category were selected. These six triples

were distributed across the six tasks so that each triple appeared once and only once in the set of tasks presented to a single child during one condition of the interview. In an attempt to control for number triple effect upon a particular task, six sets of six tasks each were generated using a Latin Square design. These six sets were randomly, but uniformly distributed across the population so that each one of the thirty-six tasks generated by this procedure was used approximately the same number of times. The actual number triples used in the study are listed in Table 3.

Table 3

Number triples used in problem tasks

Smaller numbers		Larger numbers	
2 + 3 = 5	2 + 4 = 6	3 + 8 = 11	4 + 7 = 11
2 + 5 = 7	2 + 6 = 8	5 + 7 = 12	4 + 9 = 13
3 + 4 = 7	3 + 6 = 9	6 + 8 = 14	6 + 9 = 15

During a given interview, each child was individually interviewed with responses coded on an electronically scored sheet. Ten different interviewers were utilized after each had received intensive training using both live and video taped subjects for practice. Procedures suggested by Frick & Semmel (Note 15) were utilized to determine reliability. Inter-interviewer reliability was determined to be .95 and intra-interviewer reliability to be an average of .96 with individual reliabilities ranging from .94 to 1.00.

At each interview the six tasks were scheduled to be presented under four different conditions. These conditions arose from the crossing of the two major variables being examined in this paper -- number size and availability of manipulative materials. When manipulative materials were provided, the interviewer presented to the subject for his/her use approximately 30 2 cm cubes, about half of which were blue and the other half orange. No subject was required to use the cubes when they were available. The four conditions are represented in Figure 1 along with the labels assigned to those conditions.

Figure 1

Representation of the four interview conditions

Availability of manipulatives (cubes)

Size of numbers	with	without
smaller	b+	b-
larger	c+	c-

The interview was divided into two parts with the administration of the six problems under the b+ condition followed by six problems under the b- condition being the first part. On a subsequent day, the second part consisted of the six problems under the c+ condition followed by the six problems under the c- condition. Each part lasted about 15 to 20 minutes with second part (the larger numbers) taking a slightly longer time. Problems were always read in the same order and when necessary, an individual problem was re-read as many times as required.

Although a maximum of 24 tasks could be administered in an interview, such was not the general rule. If a child was experiencing difficulty or was exhibiting confusing or useless behaviors, the interview was terminated early, either in the middle of a particular condition or at the end of the six problems under one condition. Since the ordering of the conditions -- b+, b-, c+, c- -- was assumed to conform to levels of difficulty (subsequent data confirmed this assumption to be correct), the termination rule was predicated on the further assumption that if a child was experiencing serious problems at any time, he/she would have similar or more difficult problems under later interview conditions.

Interviews were administered at five different times according to the schedule shown in Table 4. As indicated earlier, different numbers of

Table 4

Schedule of individual interviews

Interview #	Time of administration	# of children interviewed
1	October 1978	144
2	January 1979	150
3	May 1979	150
4	September 1979	99
5	January 1980	96

subjects were involved in each interview. These numbers are also shown in Table 4. Schedules for individual children were established so that a child would not receive the same set of tasks with the same number triples twice in a row. For example, if a particular set was used under the b+ condition, that set would not be used under the b- condition at the same interview time. In fact, care was exercised so that the same number triple would not be used at any two consecutive interview administrations. Therefore, at least eight months would intervene between times when a child would have a particular number triple appear in a particular addition or subtraction task.

Scoring procedures. This paper is concerned with an analysis of pupils' problem solving strategies and not with the fact that they got a problem correct or not. In order to contrast addition problems with subtraction problems where strategies are different and even to contrast different types of subtraction problems where the strategies are also potentially different (See Carpenter, et al., Note 11; Carpenter & Moser, Note 13), a scaling of strategies was established. The scaling is based upon the assumption that the strategies naturally group themselves as variables that are ordinal, although not necessarily interval. These strategy variables are described in the following paragraphs.

Level 0 (zero) strategies. These are strategies inappropriate to a particular problem. Included are using the wrong operation (adding instead of subtracting, or vice versa), guessing, stating one of the two given numbers in the problem, given no answer at all or exhibiting a behavior that could not be coded by the interviewer. Also included within this level are all the missing data where a task was not administered to a child because the interview had been broken off at an earlier stage.

Level 1 strategies. These strategies can best be described as direct modeling of the problem. Cubes or fingers (when numbers are appropriately small) are used to directly represent each of the sets given in the problem and then some physical action may be performed on the model sets to represent the actions or relationships also given in the problem. More specifically, these strategies are included.

Counting all: Coded only for addition problems. This involves construction of two separate sets, one for each addend that is counted out, "one, two, . . ., etc." and then counting the union of the two model sets, "one, two, . . ., etc." The union can be made all at once after the second addend is modeled or it can be formed incrementally as the child adjoins cubes one-at-a-time to the first set as he/she is modeling the second.

Separating: Coded only for subtraction problems. This involves a modeling of the given larger set, then taking away cubes (usually one-at-a-time) until a set equal to the smaller number is formed. Counting the remainder set yields the answer. Counting of all three sets (the original, the set taken away, the remainder) is effected "one, two, . . ., etc." except in some instances where the remainder set is small enough, the child can determine the size by subitizing. On rare occasions, the taking away is done differently so that the remainder set is equal to the second given number. In this case, counting the number of cubes (or fingers) taken away yields the answer.

Adding on: Coded only for subtraction problems. This strategy begins with a modeling of the given smaller set. Then that set is incremented one-at-a-time until the new model is equal in size to the larger number. Counting the number of cubes (or fingers) added on gives the answer. Often a child uses two colors of cubes to help identify the added-on cubes.

Matching: Coded only for subtraction problems. In this strategy,

two sets, each counted out, "one, two, . . . etc." are used to model the two given sets in the problem. These two are then physcally or visually placed in a one-to-one correspondence and the excess of the larger set over the smaller set is determined as the answer. This determination is made by counting, "one, two, . . ., etc." or by subitizing.

Level 2 strategies. These strategies can be characterized by their use of counting sequences. In these strategies, a counting sequence is entered at some point other than "one." The entry point will correspond to one of the given numbers in the problem or perhaps its successor or predecessor. The child's interpretation of the action or relationship in the problem is then represented by the decision to count forward or backward from that entry point. Once the counting sequence has been entered and the decision made to count forward or backward, the sequence continues one-by-one to some ending point. Where to terminate the counting sequence is determined either by the number of counting words spoken or by reason of the fact that the ending point is already known. In the first case, the number of counting words spoken is the second number given in the problem and the ending point is the answer. In the second case, the number of counting words spoken is the answer and the ending number is the second number in the problem. In either case, the problem solver has to carry out a double count or use some kind of tracking procedure to remember the number of counting words that have been spoken. Some children use an internal process while others use fingers, where the set of fingers is ordinarily constructed one-at-a-time as the consecutive counting words are being spoken. In rare instances the tracking set is constructed in advance of entering the counting sequence and then the counting is ended when the tracking set is exhausted. More specifically, these strategies are included.

Counting on: Coded only for addition problems. This strategy is identified by the child entering a counting sequence at a point corresponding to one of the addends. Counting is forward. Then the number of counting words spoken (either aloud or internally) corresponds to the second addend. The final word spoken is the answer. In the problems used in this study, the smaller addend was always read first. Thus, the decision to count on from the larger (or second) addend was a conscious one which was taken to be a potential indication of the child's realization of the commutative property of addition.

Counting up from given: Coded only in subtraction. This strategy is identified by entry in a counting sequence at a point corresponding to the smaller number (addend) given in the problem. Counting is forward. The counting sequence ends when the number corresponding to the larger number (sum or minuend) is spoken. The number of counting words spoken is the answer.

Counting down: Coded only in subtraction. This strategy is identified by entry into a counting sequence at a point corresponding to the larger given number (sum or minuend). Counting is backward. In most instances, the counting sequence ends when the number of counting words spoken corresponds to the smaller given number (addend) in the problem. The last number word spoken is the answer. On rare occasions, the counting sequence ends when the number word spoken corresponds to the second given number. Then the number of counting words spoken is the answer.

Level 3 strategies. These strategies are described as mental operations on numbers. In these strategies, numbers are seen not as elements of a counting sequence, but as some real entity that can be operated on independently. This is considered to be the most abstract level of behavior for the subjects of this study. They are translating the language of the problem directly into abstract mathematical representations and solving the problem in that realm. No paper and pencil was provided in the interview situations, so that all such work was done mentally. Two specific strategies are included.

Use of number facts: Coded for both addition and subtraction, this strategy is usually marked by an almost instantaneous correct response by the child. When queried by the interviewer as to how the answer was determined so quickly, the child would normally respond to the effect, "I just know that." No differentiation was made if the child indicated that the known fact was an addition fact or a subtraction fact.

Heuristic: Here the answer is determined by some mental manipulation related to another known fact. For example, if the problem deals with the sum of 5 and 7, the child might say the answer was 12 because "5 plus 5 is 10 and 2 more makes 12" or because "6 and 6 is 12 and so 5 and 7 is also 12." The assumption in the latter case is that some compensatory mechanism is working where the 1 taken from the one 6 to yield 5 is then given to the other 6 to make 7. (Or maybe the compensation goes the other way to take the 1 from 7 to make a 6 and then given to the 5 to make the other 6.)

Using these criteria, all categorical responses of subjects, or missing data, were then quantified by assignment of the scores 0, 1, 2, or 3. These scores were then subjected to the analyses described in the next section.

III. Results

The data presented in this section are not longitudinal in the strictest interpretation of that word. Data are presented from five different points in time but individual subjects are not followed over that period. Rather, group data are utilized.

In order to assess the effect of the major variables of number size and availability of manipulative materials, contrasts are made between pupil performance under the four interview conditions described earlier. By holding number size constant, contrasts between performance on b+ and b- tasks as well as c+ and c- tasks can give an indication of the effect of presence of manipulatives. In a similar fashion, by holding presence or absence of manipulatives constant, contrasts can be made to assess the effect of number size. For this analysis, contrasts are made between performance of b+ and c+ as well as b- and c-.

Using the scaled variables 0, 1, 2, and 3 for the strategies permits comparisons to be made between the six different problem types. Since these variables are essentially ordinal, certain statistical analyses are

inappropriate. Consequently, the analyses presented will be descriptive in nature.

Addition results. The nature of the results given here is consistent with those reported elsewhere (Carpenter, et al., Note 11; Carpenter & Moser, Note 6). Although one addition task (Task #1) has an "action" structure while the other (Task #4) has a "static" structure, little or no difference was found in children's performance or choice of strategy on these two problems. In fact, the overall pattern of response, no matter how it is characterized, is remarkably similar for the two. Therefore, the data are combined into a single table, Table 5. The data are also presented graphically in Figure 2.

Figure 2 & Table 5 about here

Considering first the mean scale scores, the increase in those values from Interview #1 when the children were beginning first graders to Interview #5 when they are mid-year second graders demonstrates the gradual but marked growth in the use of more sophisticated strategies. The low mean values at Interview #1, especially in the c+ and c- conditions, can be attributed to the fact that many of the first interviews were terminated before children got to those conditions, resulting in no data at all. The higher mean scores at Interview #5 reflect the fact that many children are using recall of known number facts to solve the verbal problems.

The relative usage of mental operations can be seen by considering an excerpt from Table 5, which when taken in its entirety can be confusing because of its overwhelming amount of information that is multi-dimensional in nature. If we consider simply the number size factor, we note the expected result that

	Interview #	1	2	3	4	5
Number size	smaller	10	21	37	46	58
	larger	2	6	12	13	40

the greater frequency occurs with smaller numbers regardless of interview time. A somewhat surprising result is the relatively constant growth of use of facts with the smaller numbers, especially between Interviews #3 and #4 which were the summer months when no instruction took place. The effect of instruction with larger numbers is very noticeable between Interviews #4 and #5. Although not discernable from these data, it can be reported that almost all cases of use of heuristic strategies occurred in the larger number

conditions. Inspection of Table 5 also reveals the fact that, although not very pronounced, there was always a larger incidence of use of mental operations in the conditions when manipulatives were absent rather than when they were present.

Strategy categories 1 and 2 both embody the use of some type of counting by the child. In the first case, the counting is done on and with physical objects used as models while in the second case, the counting is done essentially without objects. The results in these categories suggest an interactive effect of the two factors involved in the four interview conditions. Again, excerpts from Table 5 should prove helpful in examining the results. In considering counting with objects the first two interviews indicate that

Counting with objects (Strategy "1")					Counting without objects (Strategy "2")				
Interview#	b+	b-	c+	c-	Interview#	b+	b-	c+	c-
1	53	42	49	19	1	13	14	9	16
2	54	35	51	18	2	13	19	21	28
3	33	16	47	14	3	26	34	33	53
4	29	13	40	14	4	20	26	38	47
5	11	9	15	2	5	30	26	43	42

children behaved fairly much the same for conditions b+, b-, and c+ where fingers were used in place of the objects as devices for modeling in the b- conditions. By the third interview, there is a distinct shift in the pattern for the b- condition and it now is appearing to be similar to the c- condition. The very low incidence of counting with objects in Interview #5 is dramatic evidence of the effect of instruction and of the growing maturity of the children. Most are apparently eschewing the use of direct modeling, either with cubes or fingers, and choosing more sophisticated strategies.

When we consider counting without objects, the relatively small frequencies in Interview #1 are indicative of the fact that most first graders either do not have more sophisticated counting strategies in their repertoire or, if they do have them, do not have the ability to apply those skills in solving verbal problems. The pattern of frequency of use under the two smaller-number-size conditions (b+ and b-) shows that counting without objects never gets very large. One would surmise that children move rapidly into use of known number facts. On the other hand, there is a growth in the use of this type of counting under the larger-number-size conditions (c+ and c-), at least up to the end of first-grade (Interview #3) and the beginning of second grade (Interview #4). Apparently during the latter half of first-grade, children in this study

have learned to use counting-on strategies to solve addition problems. Whether this is taught or invented (cf. Resnick, Note 16) is not certain.

Of particular interest is the contrast between the two types of counting at the times of Interviews #3 and 4. When objects are present (b+ and c+), there is, quite naturally, a high percentage of counting with objects. At the same time, when objects are not present (b- and c-) the majority of usage shifts to counting without objects. What this seems to suggest is that many children have the ability to use more sophisticated counting strategies but do not choose to use them when objects are present. Rather they opt to fall back on a simpler, more primitive strategy. In a sense, then, the presence of manipulatives for some children stands in the way of their using a more advanced and efficient strategy.

Subtraction results. The four subtraction tasks cannot be grouped together as easily as the two addition tasks. In much the same manner as in an earlier pilot study (Carpenter, et al., Note 11) children's strategy choice appeared to be strongly determined by the semantic structure of the individual problem. The Separating problem (Task #2) was solved most predominantly by subtractive strategies of separating from or counting down. The Joining, missing addend problem (Task #6) evoked the additive strategies of adding on or counting up from given. The Comparison problem (Task #5) was the only one where the matching strategy was used in a significant way, and then only when manipulatives were present. The Part-Part-Whole, missing addend (Task #3) is the most ambiguous in terms of its semantic structure. This fact was reflected by the widest variety of strategies employed.

Despite the differences in the four problems there were some general patterns of strategy usage that emerged. These patterns were similar to some of those observed for the two addition problems. The mean scale scores increased over the year and a half time period, again reflecting the growth of children's ability to use more sophisticated strategies. The use of mental operations increased in frequency over time. For the smaller-number-size conditions, most strategies were use of number facts while the occurrence of heuristic strategies was restricted to the larger-number-size conditions. As with addition, there was a greater frequency of use of number facts with the smaller number problems than the larger. Also, more mental operation strategies usage occurred when manipulatives were absent than when they were present.

An interesting observation gained from anecdotal evidence deals with the

use of number facts. In the later interviews when fact usage was becoming much more prevalent, interviewers would query the child on how the answer was known so quickly (which was usually taken as evidence that a number fact was being used). The child would frequently respond, "I just know that *a* plus *b* is *c*." In other words, the child would respond with particular addition fact, even though the task was a subtraction problem.

Overall, the contrast between use of the different kinds of counting strategies paralleled those observed for addition. When manipulatives were present, the simpler modeling and counting was used whereas, when manipulatives were absent, the shift by some children to the more sophisticated use of counting sequences became evident. However, this shift was not as predominant, nor did it occur as quickly as it did for the addition problems.

The scaling of strategies employed in this paper provides some indication of the general difficulty level of the various problems. For example, a large incidence of the "0" strategy suggests that many children could not respond with a correct or appropriate strategy, perhaps because they did not understand the problem, or even if they did understand it, they still did not know how to go about solving it. Inspection of the data demonstrates the oft found result that addition problems are easier than subtraction problems. One exception to this was the Joining, missing addend problem. (Refer to Table 2 for the exact wording.) Performance on this task was almost the same as the two addition tasks. The other "action" subtraction problem, the Separating (Task #2) problem, appeared to produce a higher level of performance (greater use of more sophisticated strategies, and, although not reported in the data tables, more correct answers) than the two "static" problems (Tasks #3 and 5).

Each of the four subtraction problems has some results of interest that are specific to that individual problem. These will be briefly discussed in the following paragraphs.

Separating problem (Task #2). Data for this problem are presented in Table 6 and are visually represented in Figure 3.

Figure 3 & Table 6 about here

The use of number facts and heuristics does not become a dominant factor until the fifth interview. There the statistic of note is the much larger incidence of use in the b- rather than in the b+ condition. Such a large difference was not evident in the addition problems. Children seemed to

persist in the use of direct modeling and counting strategies for a much longer period of time than was the case for addition. When manipulatives were present, this is most obvious. When manipulatives were absent, they did not turn to the more sophisticated counting sequence strategies in any great way. This may be explainable by the fact that the most appropriate counting sequence strategy for this task (at least in terms of fitting the problem's semantic structure) is a backwards, counting down strategy. Backwards counting is harder for children than forward counting. Consequently, they resorted to finger models, even in the larger-number-size condition, or to an attempt to use a mental operation.

Part-Part-Whole, missing addend problem (Task #3). Data for this problem are presented in Table 7 and Figure 4.

Figure 4 & Table 7 about here

One of the most notable features of this problem is the very low incidence of use of counting sequences (strategy category "2") throughout all five interviews with the exception of the c- condition in the last three interviews where the frequency was about 25%. Part of this result may be traceable to the way in which the numbers appear in the wording of the problem. The largest number, the minuend, is read before the smaller number, the subtrahend. Taking the numbers in this order, the inclination might be to use a backwards counting sequence. As indicated earlier, backwards counting is difficult. Faced with a choice, the child may realize that since some alternative strategy must be chosen (to avoid this difficulty), it is just as easy to choose a much simpler strategy -- direct modeling.

Comparison problem (Task #5). Data for this problem are presented in Table 8 and Figure 5.

Figure 5 & Table 8 about here

The semantic structure of this problem appears to have a major effect upon the results. The static relationship between the sets described does not immediately suggest how to solve the problem. Further, the two sets are disjoint and are never combined to be subsets of a third, larger superset. Rather, the larger of the given sets must be partitioned into two parts so that one part matches the size of the other given set. As such, the seemingly most appropriate strategy, in terms of fitting the semantic structure, is

matching which can only be done with manipulatives present. The one exception that can be done without objects and with fingers is the problem where the given numbers are 3 and 5. Thus, in the manipulatives absent conditions (b- and c-), the children who may have decided that matching is a useful strategy are literally forced to re-examine the semantic structure and choose a different strategy. Seemingly, many children were unable to do this. The very large incidence of the "0" strategy in the b- and c- conditions up to and including the fourth interview is the evidence for this assertion. In fact, many subjects recited one of the given numbers in the problem as the answer -- a totally inappropriate strategy. The other evidence for the previous assertion is the very low frequencies of the direct modeling strategy in the c- condition during all five interviews, particularly when those frequencies are contrasted with their counterparts in the c+ condition. Of interest in this same vein is the gradual but steady increase of use of counting sequences in the c- condition as children learned the use of those more sophisticated counting strategies as the alternative choice to the direct modeling (matching) that is impossible in the c- condition.

Part-Part-Whole, missing addend problem (Task #6). The data for this problem are presented in Table 9 and Figure 6.

Figure 6 & Table 9 about here

As indicated earlier, this problem proved to be the easiest of the four subtraction problems, and in fact, is very similar to the two addition problems in terms of overall performance. The wording of the problem very clearly conveys the structure, making it a simple matter for children to choose the appropriate strategy. This problem is solved by additivie methods, either counting up from the given smaller number or by using an addition fact. The spectacular increase in use of these more advanced strategies from Interview #4 to Interview #5 indicates that children have learned this strategy quite well. Anecdotal evidence has shown that counting up can be executed very rapidly, particularly when the number of upward counts is only 2 or 3 as is the case in the smaller-number-size conditions. Sometimes the counting is so rapid that it is virtually impossible for an interviewer to distinguish between counting and recall of a known number fact. A plausible hypothesis is that many children never completely commit some facts to long term memory, but rather use the rapid counting-on algorithm to determine what is, even to them,

a "known" fact. Thus, a child would respond that he/she "just knew a fact" even when counting took place. This indistinguishability of rapid counting-on from recall of number facts may well be particularly prevalent in this problem because the wording is so unambiguous that many children can begin processing a counting-on algorithm before the complete problem has even been posed.

IV. Discussion

The results presented in this paper give a rather complete picture of the development of problem solving abilities of the subjects involved over a period of a year and a half in terms of the six problem types used in the study. From the beginning baseline interviews conducted when children were still in their first month of first-grade to the fifth interview when they had completed half of second grade, the portrait painted is one of substantial growth. In the early stages, many children could not solve the problems, particularly in the more difficult conditions when no manipulatives were present and when numbers were larger. When problems were solved, the great majority of successful solvers used a somewhat simple strategy of direct modeling of sets and relationships with physical objects or with fingers. As time passed and instruction was absorbed, the children moved into the use of more sophisticated strategies such as counting sequences or use of known number facts or heuristics. It should be noted that the data come from group statistics and individual student progress over time has not been analyzed at this time. Whether the sequence of progress through the several levels of sophistication of strategy is a linear, invariant one that all students must pass through is unknown but certainly a subject for further investigation.

Although there are some variations from problem to problem, the general trend of the results indicates that presence of manipulatives does have an effect. When manipulatives are present in the interview condition, the tendency is for children to use them as part of the direct modeling strategy even though there is evidence to indicate that many of the children are capable of using more sophisticated counting or mental operation strategies. This tendency does diminish sharply by the middle of second grade, especially in the smaller-number-size conditions where the children opt to use sophisticated counting or number facts. Variation exists here as well. In some of the harder subtraction problems, the tendency to remain with the direct modeling strategy for a longer period of time is evident.

The number size effect was predictable. In smaller-number problems,

children used the more sophisticated strategies at an earlier point in time and with more frequency than with the larger-number problems. Here, too, the variability of problem types was a factor. The movement to the more sophisticated strategies occurred with greater frequency at an earlier time with the two addition problems and the Joining, missing addend subtraction problem than with the other three subtraction problems.

The data seem to indicate that the children are near to "topping out" on the smaller number problems. No further interviews with those problems are contemplated with the present subjects of this study. The larger-number problems will be continued in later interviews to be conducted in May 1980 and September 1980. Whether the same results will occur with different populations or with differently worded problems or with problems having a different semantic structure is open to question. It is the intent of the Mathematics Work Group of the Wisconsin Research and Development Center to investigate such questions in its future research endeavors.

Reference Notes

1. Vergnaud, G. A classification of cognitive tasks and operations of thought involved in addition and subtraction problems. Paper presented at the Wingspread Conference, Racine, Wisconsin, November 1979.
2. Heller, J. I. & Greeno, J. G. Semantic processing in arithmetic word problem solving. Paper presented at the meeting of the Midwestern Psychological Association, Chicago, May 1978.
3. Riley, M. S. & Greeno, J. G. Importance of semantic structure in the difficulty of arithmetic word problems. Paper presented at the meeting of the Midwestern Psychological Association, Chicago, May 1978.
4. Nesher, P. S. & Katriel, T. Two cognitive modes in arithmetic word problem solving. Paper presented at the second annual meeting of the International Group for the Psychology of Mathematics Education, Osnabrück, West Germany, September 1978.
5. Moser, J. M. Young children's representation of addition and subtraction problems (Theoretical Paper No. 74). Madison: Wisconsin Research and Development Center for Individualized Schooling, May 1979.
6. Carpenter, T. P. & Moser, J. M. An investigation of the learning of addition and subtraction (Theoretical Paper No. 79). Madison: Wisconsin Research and Development Center for Individualized Schooling, November 1979.

7. Ibarra, C. G. & Lindvall, C. M. An investigation of factors associated with children's comprehension of simple story problems involving addition and subtraction prior to formal instruction on these operations. Paper presented at the annual meeting of the National Council of Teachers of Mathematics, Boston, April 1979.

8. Ginsburg, H. P. & Russell, R. L. The development of mathematical thinking: Social class, race and family status. Unpublished manuscript, 1979.

9. Steffe, L. P., Spikes, W. C., & Hirstein, J. J. Quantitative comparisons and class inclusion as readiness variables for learning first grade arithmetical content. Athens, GA: The Georgia Center for the Study of Learning and Teaching Mathematics, University of Georgia, 1976.

10. Riley, M. S. The development of children's ability to solve arithmetic word problems. Paper presented at the annual meeting of the American Educational Research Association, San Francisco, April 1979.

11. Carpenter, T. P., Hiebert, J., & Moser, J. M. The effect of problem structure on first-graders' initial solution processes for simple addition and subtraction problems (Technical Report No. 516). Madison: Wisconsin Research and Development Center for Individualized Schooling, October 1979.

12. Carpenter, T. P. The effect of initial instruction on first-grade children's solution processes for basic addition and subtraction problems. Paper presented at annual meeting of American Educational Research Association, Boston, April 1980.

13. Carpenter, T. P. & Moser J. M. The development of addition and subtraction concepts in young children. Paper presented at the third annual meeting of the International Group for the Psychology of Mathematics Education, Warwick, England, July 1979.

14. Kouba, V. L. & Moser, J. M. Development and validation of curriculum units related to initial sentence writing (Technical Report No. 522). Madison: Wisconsin Research and Development Center for Individualized Schooling, October 1979.

15. Frick, T. & Semmel, M. I. Observational records: Observer agreement and reliabilities. Paper presented at the annual meeting of the American Educational Research Association, Chicago, April 1974.

16. Resnick, L.B. The role of invention in the development of mathematical competence. Paper presented at the Workshop on Children's Mathematical Learning, Learning Research and Development Center, Pittsburgh, September 1978.

References

Greeno, J.G. Preliminary steps towards a cognitive model of learning primary mathematics. In K.C. Fuson & W.E. Geeslin (Eds.) Explorations in the modeling of the learning of mathematics. Columbus: ERIC/SMEAC Center for Science, Mathematics and Environmental Education, 1979.

Groen, G.J. & Parkman, J.M. A chronometric analysis of simple addition. Psychological Review, 1972, 79, 329-343.

Groen, G.J. & Poll, M. Subtraction and the solution of open sentence problems. Journal of Experimental Child Psychology, 1973, 16, 292-312.

Grouws, D.A. Open sentences: Some instructional considerations from research. Arithmetic Teacher, 1972, 19, 595-599.

Grunau, R.V. Effects of elaborative prompt condition and developmental level on the performance of addition problems by kindergarten children. Journal of Educational Psychology, 1978, 70, 422-432.

Hebbeler, K. Young children's addition. The Journal of Children's Mathematical Behavior, 1977, 1, 108-121.

Hendrickson, A.D. An inventory of mathematical thinking done by incoming first-grade children. Journal for Research in Mathematics Education, 1979, 10, 7-23.

Hirstein, J.J. Children's counting in addition, subtraction, and numeration contexts (Doctoral dissertation, University of Georgia, 1978). Dissertation Abstracts International, 1979.

Jerman, M. Problem length as a structural variable in arithmetic problems. Educational Studies in Mathematics, 1973, 5, 109-123.

Jerman, M. & Rees, R. Predicting the relative difficulty of verbal arithmetic problems. Educational Studies in Mathematics, 1973, 4, 306-323.

Jerman, M. & Sanford, M. Linguistic and computational variables in problem solving in elementary mathematics. Educational Studies in Mathematics, 1974, 5, 317-362.

Lindvall, C.M. & Ibarra, C.G. Incorrect procedures used by primary grade pupils in solving open addition and subtraction sentences. Journal for Research in Mathematics Education, 1980, 11, 50-62.

Linville, W.J. Syntax, vocabulary and the verbal arithmetic problems. School Science and Mathematics, 1976, 76, 152-158.

Nesher, P.S. Three determinants of difficulty in verbal arithmetic problems. Educational Studies in Mathematics, 1976, 7, 369-388.

Nesher, P.S. & Teubal, E. Verbal cues as an interfering factor in verbal problem solving. Educational Studies in Mathematics, 1974, 6, 41-51.

Resnick, L.B., Wang, M.C. & Kaplan, J. Task analysis in curriculum design. Journal of Applied Behavioral Analysis, 1973, 6, 679-710.

Romberg, T.A., Harvey, J.G., Moser, J.M. & Montgomery, M.E. Developing Mathematical Processes, Chicago: Rand McNally & Co., 1974.

Schwartz, A.N. Assessment of the mathematical concepts of five-year-old children. Journal of Experimental Education, 1969, 37, 67-74.

Searle, B.W., Lorton, P.J. & Suppes, P. Structural variables affecting CAI performance on arithmetic word problems of disadvantaged and deaf students. Educational Studies in Mathematics, 1974, 5, 371-384.

Vergnaud, G. & Durand, C. Structures additives et complexité psychogénétique. La Revue Francaise de Pedagogie, 1976, 36, 28-43.

Weaver, J.F. Some factors associated with pupils' performance levels on simple open addition and subtraction sentences. Arithmetic Teacher, 1971, 18, 513-519

Williams, A.H. Mathematical concepts, skills, and abilities of kindergarten entrants. Arithmetic Teacher, 1965, 12, 262-268.

Woods, S.S., Resnick, L.B. & Groen, G.J. An experimental test of five process models for subtraction. Journal of Educational Psychology, 1975, 67, 17-21.

Table 5

Percentage of Children Using Strategies on Tasks 1 and 4 (The Two Addition Tasks) Under Various Problem Conditions

Interview # Date	Strategy (scaled scores)	b+	b-	c+	c-	Number size b	Number size c	Manipulatives +	Manipulatives -
	0	25	32	40	64	28	52	32	48
1	1	53	42	49	19	48	34	51	31
October 1978	2	13	14	9	16	13	12	11	15
[n = 144]	3	9	12	3	1	10	2	6	7
	(mean scale score)	(1.06)	(1.06)	(.74)	(.55)	(1.06)	(.65)	(.90)	(.81)
	0	14	24	22	48	19	35	18	36
2	1	54	35	51	18	44	35	53	26
January 1979	2	13	19	21	28	16	25	17	24
[n = 150]	3	19	22	5	6	21	6	12	14
	(mean scale score)	(1.38)	(1.41)	(1.10)	(.92)	(1.39)	(1.01)	(1.24)	(1.17)
	0	8	10	7	22	9	14	7	16
3	1	33	16	47	14	25	30	40	15
May 1979	2	26	34	33	53	30	43	33	44
[n = 150]	3	33	40	13	11	37	12	23	26
	(mean scale score)	(1.85)	(2.04)	(1.52)	(1.54)	(1.94)	(1.53)	(1.69)	(1.79)
	0	7	12	10	20	10	15	9	16
4	1	29	13	40	14	21	27	35	14
September 1979	2	20	26	38	47	23	43	29	37
[n = 99]	3	44	48	12	15	46	13	28	32
	(mean scale score)	(2.00)	(2.11)	(1.51)	(1.64)	(2.06)	(1.58)	(1.76)	(1.88)
	0	4	4	7	13	4	10	5	8
5	1	11	9	15	2	10	8	13	6
January 1980	2	30	26	43	42	28	42	36	34
[n = 96]	3	55	61	35	44	58	40	45	52
	(mean scale score)	(2.36)	(2.43)	(2.07)	(2.17)	(2.40)	(2.12)	(2.22)	(2.30)

Note. Strategy scale: 0 - No response or inappropriate; 1 - Direct modeling; 2 - Use of counting sequence; 3 - Mental operations

Number size: b - smaller; c - larger

Manipulatives: + : present; - : absent

Figure 2

Tasks 1&4 (the two addition tasks)

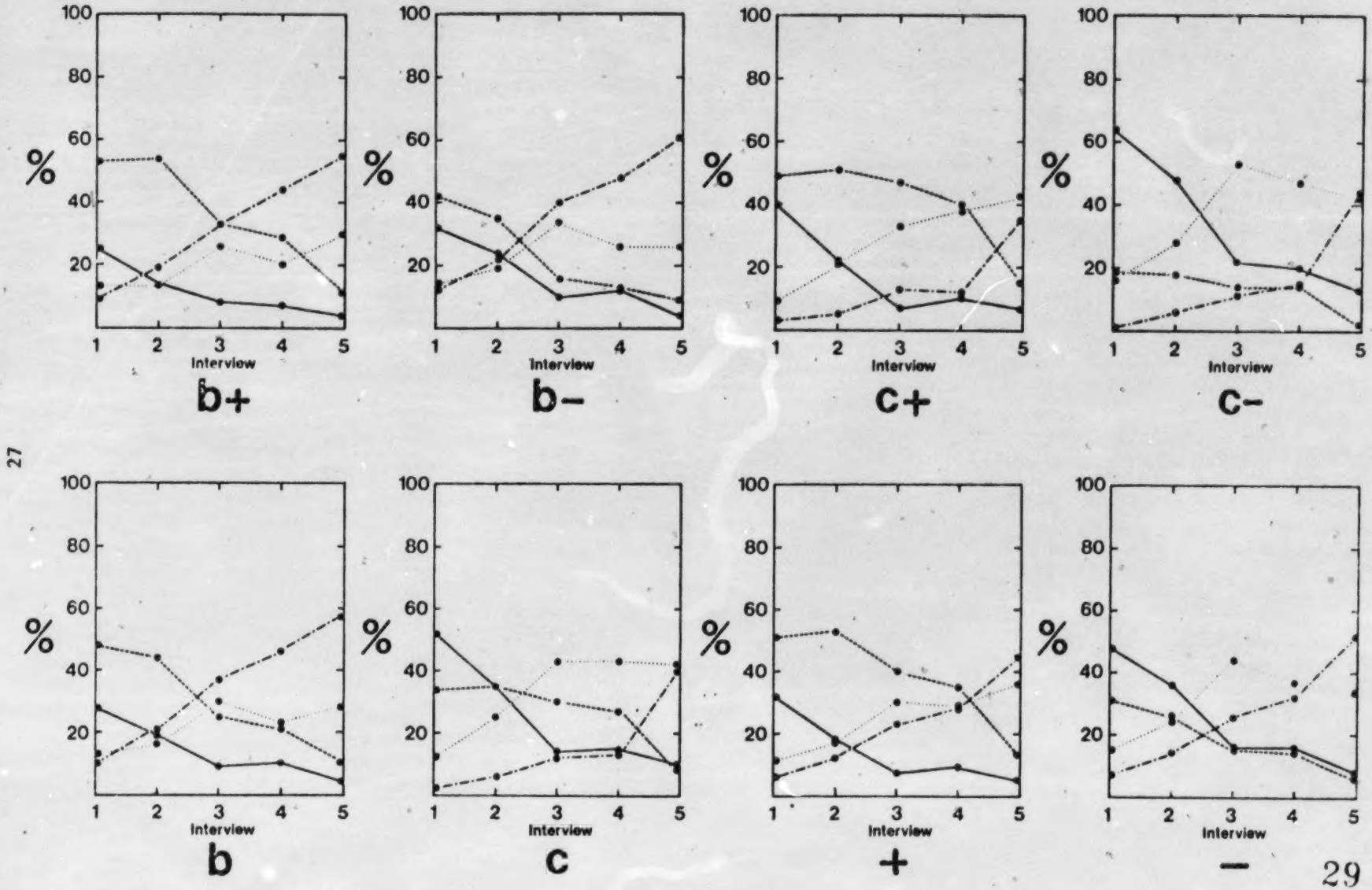

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Table 6

Percentage of Children Using Strategies on Task 2 Separating (Subtraction) Under Various Problem Conditions

Interview # Date	Strategy (scaled scores)	b+	b-	c+	c-	Number size b	Number size c	Manipulatives +	Manipulatives -
	0	36	44	40	69	40	54	38	56
1	1	52	38	57	24	45	40	55	31
October 1978	2	6	9	2	6	8	4	4	8
[n = 144]	3	6	10	1	1	8	1	3	6
	(mean scale score)	(.81)	(.85)	(.65)	(.40)	(.83)	(.53)	(.73)	(.63)
	0	17	42	24	67	30	46	21	55
2	1	63	40	69	19	51	44	66	29
January 1979	2	5	5	3	11	5	7	4	8
[n = 150]	3	15	13	3	3	14	3	9	8
	(mean scale score)	(1.17)	(.89)	(.85)	(.49)	(1.03)	(.67)	(1.01)	(.69)
	0	9	13	10	35	11	23	10	24
3	1	57	44	65	25	50	45	61	34
May 1979	2	9	16	15	25	12	20	12	20
[n = 150]	3	26	27	9	15	26	12	18	21
	(mean scale score)	(1.52)	(1.56)	(1.23)	(1.20)	(1.54)	(1.22)	(1.38)	(1.38)
	0	13	17	17	30	15	24	15	19
4	1	54	36	59	26	45	42	56	31
September 1979	2	5	10	9	26	8	18	7	18
[n = 99]	3	28	36	15	17	32	16	22	27
	(mean scale score)	(1.48)	(1.66)	(1.22)	(1.30)	(1.57)	(1.26)	(1.35)	(1.48)
	0	10	8	13	18	9	15	11	13
5	1	36	20	35	16	28	26	36	18
January 1980	2	11	13	22	26	12	24	17	19
[n = 96]	3	42	59	30	41	51	35	36	50
	(mean scale score)	(1.84)	(2.23)	(1.70)	(1.90)	(2.04)	(1.80)	(1.77)	(2.02)

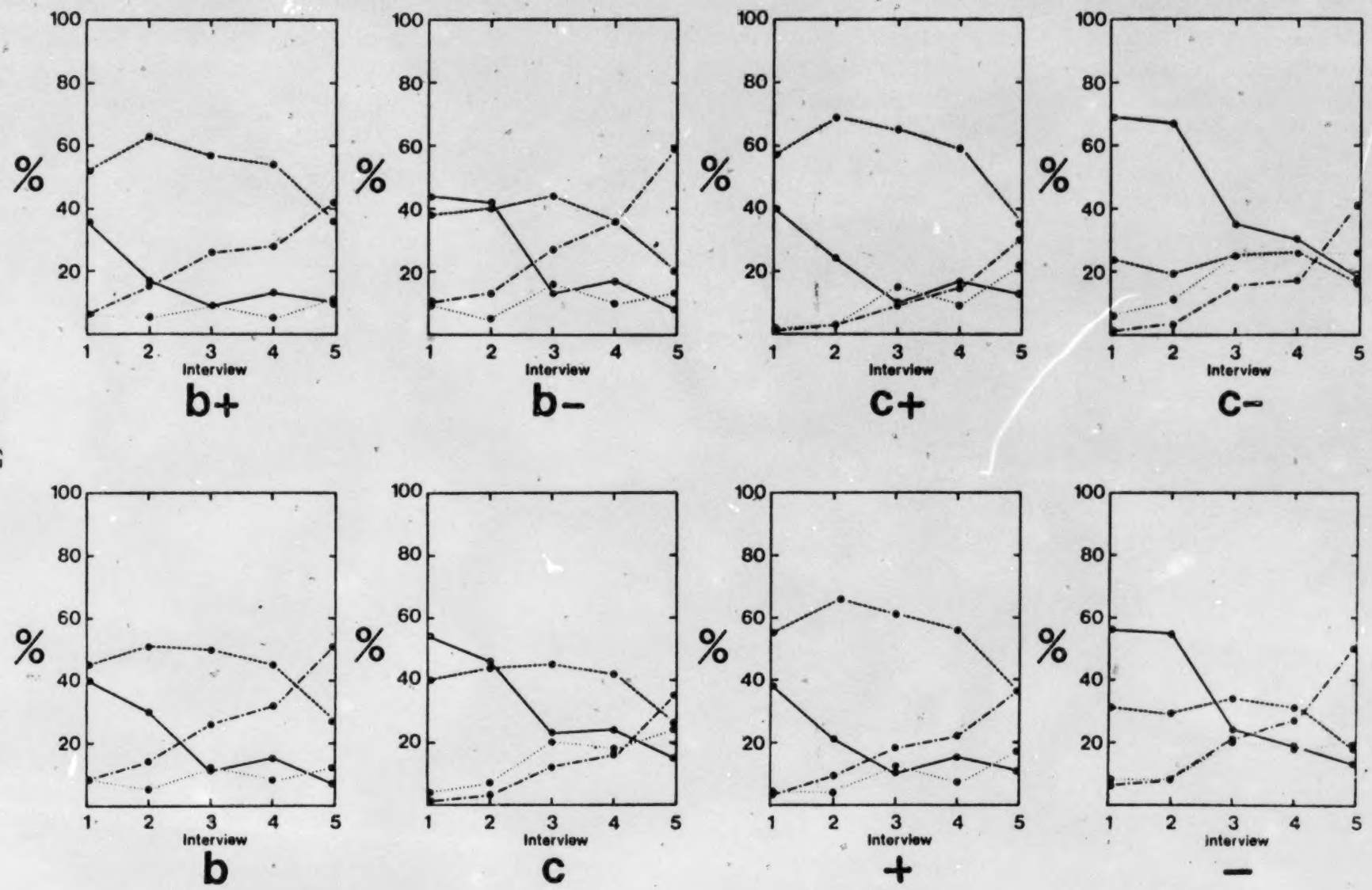
Note. Strategy scale: 0 - No response or inappropriate; 1 - Direct modeling; 2 - Use of counting sequence; 3 - Mental operations

Number size: b - smaller; c - larger

Manipulatives: + : present; - : absent

Figure 3

Task 2 Separating [subtraction]



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Strategy 0: ____ 1: ..... 2: ..... 3: _._._.

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Table 7

Percentage of Children Using Strategies on Task 3 Part-Part-Whole, Missing Addend (Subtraction) Under Various Problem Conditions

Interview # Date	Strategy (scaled scores)	b+	b-	c+	c-	Number size b	Number size c	Manipulatives +	Manipulatives -
	0	52	63	51	75	58	63	52	69
1	1	37	27	43	16	32	30	40	22
October 1978	2	6	5	3	8	6	6	5	6
[n = 144]	3	5	5	2	1	5	2	3	3
	(mean scale score)	(.64)	(.51)	(.56)	(.35)	(.58)	(.46)	(.60)	(.43)
	0	50	56	43	75	53	59	46	66
2	1	33	23	49	9	28	29	41	16
January 1979	2	5	6	4	12	6	8	5	9
[n = 150]	3	11	15	4	3	13	4	8	9
	(mean scale score)	(.78)	(.81)	(.69)	(.43)	(.80)	(.56)	(.74)	(.62)
	0	23	36	19	48	29	33	21	42
3	I	52	32	58	19	42	39	55	26
May 1979	2	5	10	13	23	11	18	9	16
[n = 150]	3	21	22	11	10	21	10	16	16
	(mean scale score)	(1.23)	(1.18)	(1.15)	(.95)	(1.21)	(1.05)	(1.19)	(1.07)
	0	20	23	22	43	22	33	21	33
4	1	41	32	51	14	37	33	46	23
September 1979	2	7	11	14	28	9	21	11	20
[n = 99]	3	31	33	12	14	32	13	22	24
	(mean scale score)	(1.49)	(1.55)	(1.16)	(1.13)	(1.52)	(1.15)	(1.33)	(1.34)
	0	19	19	13	21	19	17	16	20
5	1	28	21	36	18	24	27	32	19
January 1980	2	13	10	18	27	11	22	15	19
[n = 96]	3	41	50	33	34	45	34	37	42
	(mean scale score)	(1.75)	(1.92)	(1.72)	(1.75)	(1.84)	(1.74)	(1.74)	(1.84)

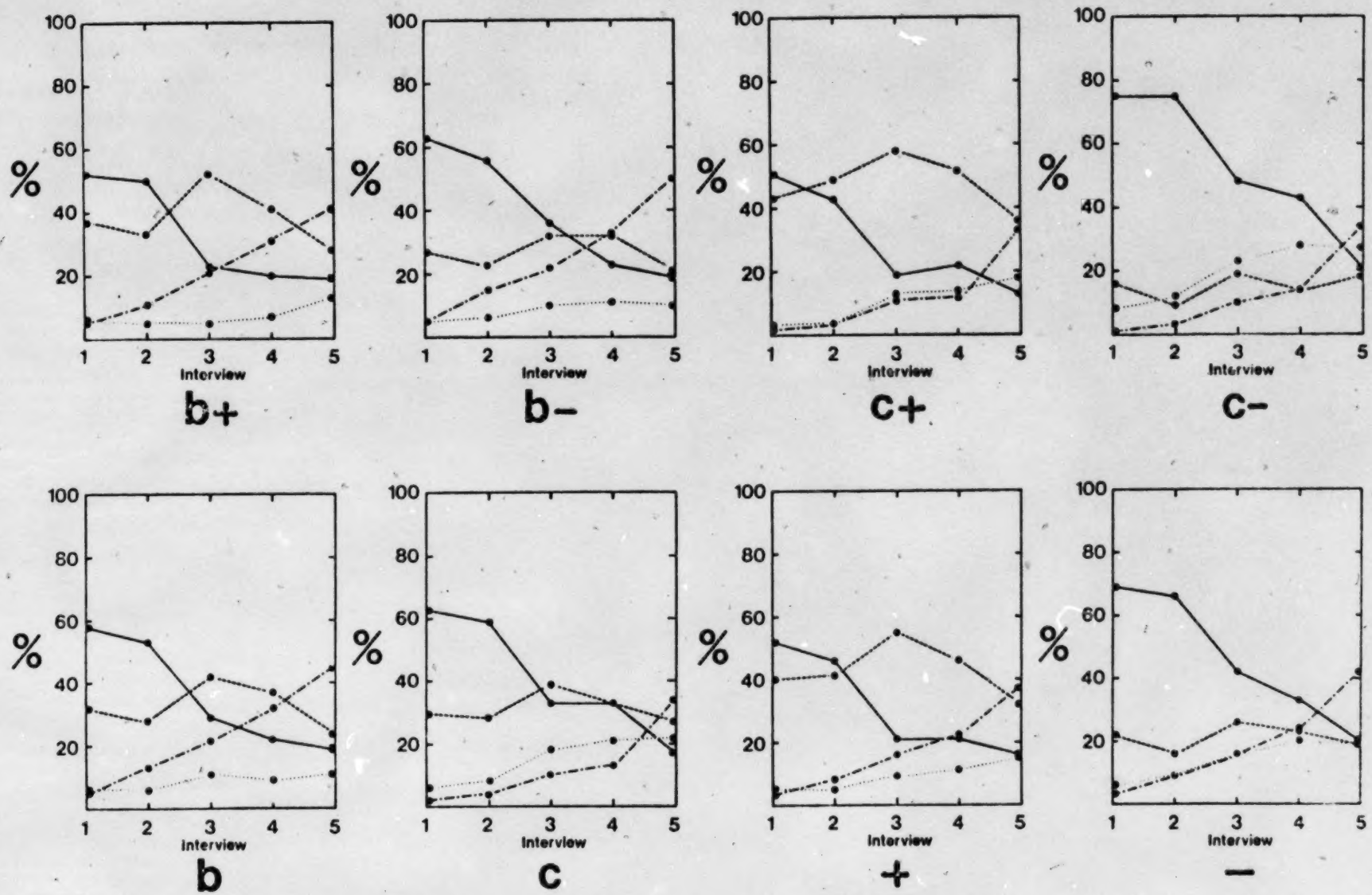
Note. Strategy scale: 0 - No response or inappropriate; 1 - Direct modeling; 2 - Use of counting sequence; 3 - Mental operations

Number size: b - smaller; c - larger

Manipulatives: + : present; - : absent

Figure 4

Task 3 Part-Part-Whole, missing addend [subtraction]



Strategy 0: ____ 1: ..... 2: ..... 3: _._._.

Table 8

Percentage of Children Using Strategies on Task 5 Comparison (Subtraction) Under Various Problem Conditions

Interview # / Date	Strategy (scaled scores)	b+	b-	c+	c-	Number size: b	Number size: c	Manipulatives: +	Manipulatives: -
	0	54	62	56	78	58	67	55	70
1	1	35	20	35	6	28	20	35	13
October 1978	2	7	13	7	14	10	10	7	13
[n = 144]	3	3	6	1	3	5	2	2	4
	(mean scale score)	(.60)	(.62)	(.53)	(.42)	(.61)	(.48)	(.57)	(.52)
	0	43	55	48	69	49	59	46	62
2	1	40	20	41	7	30	24	41	13
January 1979	2	9	16	9	18	13	13	9	17
[n = 150]	3	7	9	2	6	8	4	4	8
	(mean scale score)	(.79)	(.80)	(.65)	(.61)	(.80)	(.63)	(.72)	(.71)
	0	36	42	28	60	39	44	32	51
3	1	34	15	52	7	24	29	41	11
May 1979	2	9	20	13	24	14	15	11	22
[n = 150]	3	22	23	7	10	23	9	15	17
	(mean scale score)	(1.17)	(1.25)	(.99)	(.85)	(1.21)	(.92)	(1.08)	(1.05)
	0	28	36	27	48	32	38	28	42
4	1	25	13	42	9	19	26	34	11
September 1979	2	16	19	20	31	18	26	18	25
[n = 99]	3	30	31	10	11	31	11	20	21
	(mean scale score)	(1.48)	(1.45)	(1.13)	(1.05)	(1.47)	(1.09)	(1.31)	(1.25)
	0	16	23	24	29	19	27	20	26
5	1	26	10	27	5	18	16	27	8
January 1980	2	16	22	23	34	19	29	19	28
[n = 96]	3	43	45	26	31	44	29	34	38
	(mean scale score)	(1.85)	(1.89)	(1.51)	(1.68)	(1.87)	(1.60)	(1.68)	(1.79)

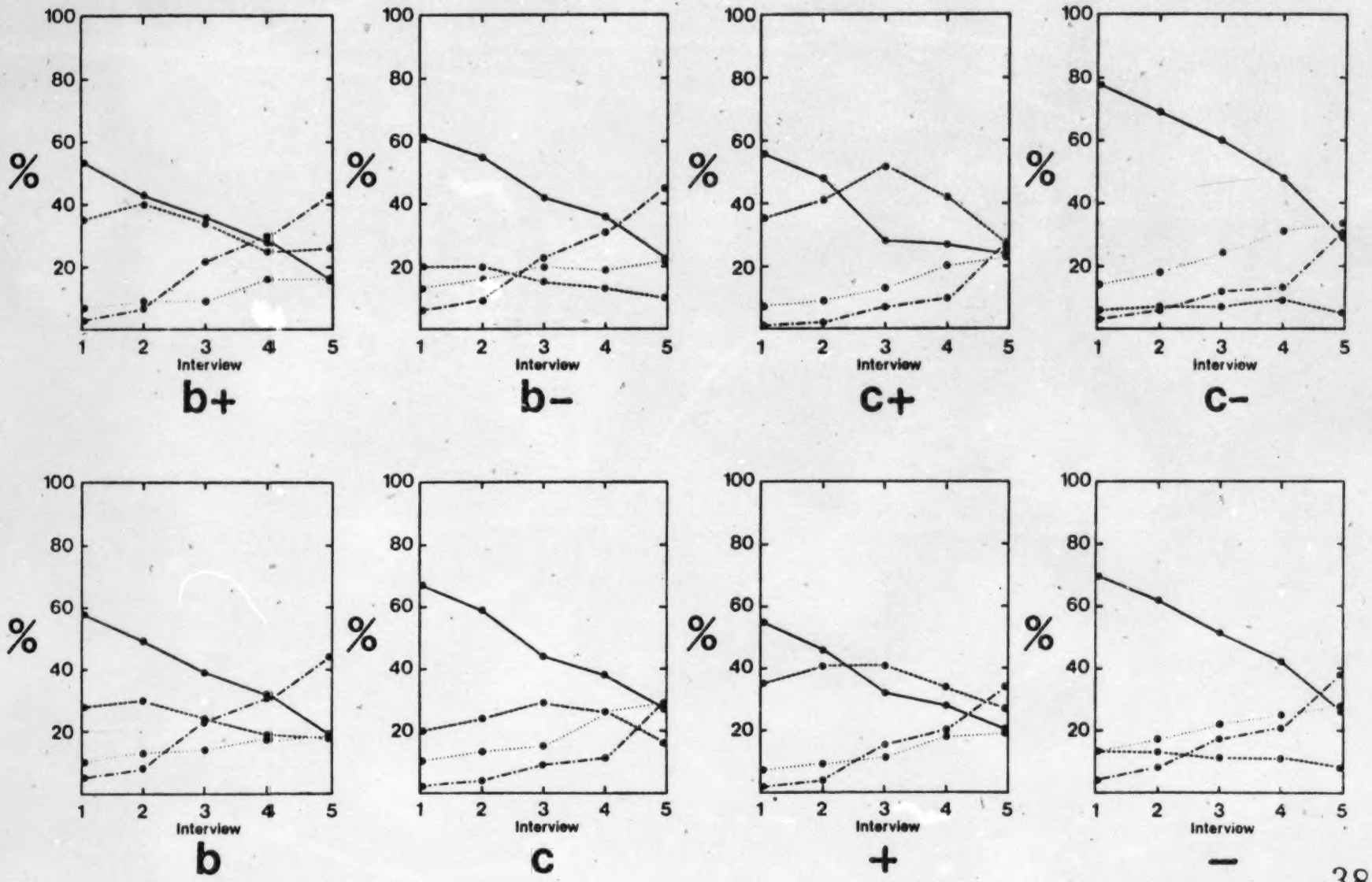
Note. Strategy scale: 0 - No response or inappropriate; 1 - Direct modeling; 2 - Use of counting sequence; 3 - Mental operations

Number size: b - smaller; c - larger

Manipulatives: + : present; - : absent

Figure 5

Task 5 Comparison [subtraction]



Strategy 0: ____ 1: ..... 2: ..... 3: _._._.

Table 9

Percentage of Children Using Strategies on Task 6 Joining, Missing Addend (Subtraction) Under Various Problem Conditions

Interview # Date	Strategy (scaled scores)	b+	b-	c+	c-	Number size b	Number size c	Manipulatives +	Manipulatives -
	0	36	35	49	73	36	61	42	54
1	1	47	37	40	7	42	23	43	22
October 1978	2	10	20	9	18	15	14	10	2
[n = 144]	3	6	7	3	2	7	2	5	5
	(mean scale score)	(.87)	(1.00)	(.66)	(.49)	(.94)	(.58)	(.77)	(.75)
	0	19	37	36	60	28	48	28	48
2	1	47	29	46	11	38	28	46	20
January 1979	2	16	16	12	26	16	19	14	21
[n = 150]	3	18	19	6	3	18	5	12	11
	(mean scale score)	(1.33)	(1.17)	(.88)	(.73)	(1.25)	(.81)	(1.11)	(.95)
	0	11	19	17	29	15	23	14	24
3	1	33	21	52	15	27	33	42	18
May 1979	2	20	26	18	44	23	31	19	35
[n = 150]	3	36	35	13	13	35	13	25	24
	(mean scale score)	(1.81)	(1.77)	(1.28)	(1.41)	(1.79)	(1.35)	(1.55)	(1.59)
	0	16	16	18	27	16	23	17	22
4	1	25	25	41	15	25	28	33	20
September 1979	2	15	23	26	44	19	30	21	34
[n = 99]	3	43	35	14	13	39	14	29	24
	(mean scale score)	(1.86)	(1.78)	(1.36)	(1.43)	(1.82)	(1.40)	(1.61)	(1.61)
	0	1	4	7	8	3	8	4	6
5	1	14	8	19	6	11	13	16	7
January 1980	2	21	26	33	50	23	42	27	38
[n = 96]	3	65	61	41	35	63	38	53	48
	(mean scale score)	(2.49)	(2.45)	(2.07)	(2.13)	(2.47)	(2.10)	(2.28)	(2.29)

Note. Strategy scale: 0 - No response or inappropriate; 1 - Direct modeling; 2 - Use of counting sequence; 3 - Mental operations

Number size: b - smaller; c - larger

Manipulatives: + : present; - : absent

Figure 6

Task 6 Joining, missing addend [subtraction]